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more or less indispensable—involves a reference to a genuine item of reality. In some sense, therefore, possibility is real. Secondly, the problem of 'alternative possibles' is left open. Analysis of the idea of possibility does not inform us whether the execution is determined or undetermined. So, having marched up to the edge of a familiar battle-field, wc may reasonably halt.

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BIOMETRY

IT is the history of all sciences that have been developed to any extent, that they pass from a period of superficial observations and speculations, through one of more careful investigation, to one in which all observations are recorded by means of numbers, and the speculations are based upon mathematical formulæ. In other words, while the expression, 'an exact science,' is, in a way, tautological, still it may fittingly be used to express the goal toward which all sciences are pushing; and their progress in that direction may be judged by the extent to which their students express their ideas in quantitative terms.

It is, at present, difficult to conceive of the science of astronomy, for example, apart from formulæ and figures. Physics and chemistry have passed from the purely qualitatively observational stage to the realms of calculus and algebra. And in the study of the evolution of living beings, it is a hopeful sign that its workers are no longer content to say that a given species varies in a certain *way,* but tell us *how much* it varies in that way. It is no longer sufficient to say that offspring, as a rule, are like the parents; but we must know how much like their parents they are. Natural selection seems to be a very important factor in evolution; but just how real a factor is it?

Large masses of individuals under certain conditions must be carefully measured, and their measurements plotted and compared with those of masses from other conditions. The numerous laws of variation which have been proposed need careful and accurate testing with the rule and balance, on populations from the field and the breeding room. "Whatever views we hold on selection, inheritance or fertility, we must ultimately turn to the mathematics of large numbers, to the theory of mass-phenomena, to interpret safely our observations. As we can not follow the growth of nations without statistics of birth, death, duration of life, marriage and fertility, so it is impossible to follow the changes of any type of life without its vital statistics.'"[1] It is with these vital statistics that the relatively new phase of investigation known as 'biometry' deals.

It is the study to which Darwin looked forward when he said,[2] "The chief point, which I am, and have been for years, very anxious about, is to ascertain whether the *young* of our domestic breeds differ as much from each other as do their parents, and *I have no faith in anything short of actual measurements and the Rule of Three.*" (The latter italics are mine.) Let us, then, consider, briefly, some of the work which has already been done along this line and the methods employed.

It seems to be generally true in physical as well as mental characters that mediocrity is the rule. If a group of individuals, taken at random from a given species, be arranged in subgroups on the basis of the varying magnitude of a certain character, it will usually be found that a great majority of these individuals fall in the classes but little removed from the middle value, while the extreme classes, *i. e.*, those having a value much greater or smaller than the middle value, contain but few individuals. When data of this sort are plotted in the manner which I will now describe we get the so-called *curve of frequencies.*

The various classes are arranged in their order of magnitude. Perpendiculars are erected on a horizontal line as a base. The lengths of these perpendiculars are proportional to the number of individuals in each of the classes, starting at the left with the class having the smallest magnitude of the character in question. The spaces between the perpendiculars should be equal, as should also the range of magnitude of each of the various classes. Now, when the tops of these perpendiculars are joined by a flowing line, we find we have a curve high in the middle (representing the large number of mediocre individuals) and tapering off at each end. For most measurements it will be found to be practically symmetrical about the class containing the largest number of individuals—the most 'fashionable' class, hence called the *mode.* This class closely corresponds to the average, or mean value of the character, and the curve itself may be represented by the formula,

$$y = \frac{n}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

which is the formula of the 'Normal Curve of Error,' first deduced by Gauss at the beginning of the last century. We may conceive that the reason so many biological measures conform to this 'law of probability' or 'error' is that among the countless agencies at work influencing the growth of a character there are as many favoring it as opposing it, and these, working in all possible combinations, give rise to a curve typical of such chance combinations. However, whatever the reason, the fact remains that this formula, or a modification of it, can be used to represent accurately the data at hand; and if this were all, it would still be of immense value. Let us see some of the things it tells us.

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[1] *Biometrica*, I., p. 3.

[2] *Life*, II., p. 51.

In the first place, it is important to determine the modes of the characters of various species at a given place and time, and compare them with similar modes observed at other places and times, in order to discover just how much evolution, if any, is going on and in what direction the species is moving.

We also have, in the shape of the curve, a measure of the variability of the organ, and can compare it with the variability of other organs, *e. g.*, to discover if secondary sexual characters are really extremely variable. When plotted on the same scale, the more variable character will give a low flat curve, while the less variable one will give a curve with short range, one in which most of the individuals are grouped in, or near, the modal class. The range is itself a measure of variability, but a very poor one. A much better one is the σ of the formula given above. It is the square root of the average of the squared departures from the mean in both directions; and is called the *standard deviation.*

But, although many measurements conform closely to this symmetrical unimodal type of curve, there are many interesting departures. The curves are often 'skew,' *i. e.*, there is a tendency to produce more individuals on one side of the mode than on the other. The chances are not equal that the favorable agencies will exactly counterbalance the unfavorable. Here we would *expect* a gradual shifting of the mode—the evolution of a species—in course of time; but, probably, time alone will tell what skewness actually does mean. It is sometimes so great—*e. g.*, in the case of the number of petals in buttercups—that there are absolutely no individuals on one side of the mode.

Again, the curve may have more than one mode. There are clearly two or more centers of stability. Does it mean that the species is splitting up? An interesting fact in this connection is that in the majority of cases where dimorphism has been investigated, the two halves are skew toward each other. There is need here for much further work.

But probably the most fruitful field of biometry is that of *correlation.* Davenport[3] defines correlated variation as 'such a relation between the magnitudes of two or more characters that any

[3] 'Statistical Methods,' 2 Ed., p. 42.

abmodality of the one is accompanied by a corresponding abmodality of the other or others.' He further states, that "the principles upon which the measure of correlated variation rests are these. When we take individuals at random we find that the mean magnitude of any character is equal to the mean magnitude of this character in the whole population. Deviation from the mean of the whole population in any lot of individuals implies a selection. If we select individuals on the basis of one character (A, called the *subject*), we select also any closely correlated character (B, called the *relative*)—*e. g.*, leg-length and stature. If perfectly correlated the index of admodolity[4] of any class of B will be as great as that of the corresponding class of A, or

$$\frac{\text{Index abmodality of relative class}}{\text{Index abmodality of subject class}} = 1.$$

"If there is no correlation, then whatever the value of the index of abmodality of the subject, that of the relative will be zero" (since the relative class is, in this case, random sampling) "and the coefficient of correlation will be

$$\frac{\text{Index of abmodality of relative class}}{\text{Index of abmodality of subject class}} = \frac{o}{m} = 0.$$

"The coefficient of correlation is represented in formulas by the letter r. We can not find the degree of correlation between two organs by measuring a single pair only; it is the correlation 'in the long run' which we must consider. Hence we must deal with masses and with averages."

This method is applicable not only to the study of the correlation of various characters—physical, physiological and psychological—as they occur in the same individual, but it has been widely used in the study of heredity as a measure of the likeness of offspring to ancestry, or the likeness between co-parentals. It furnishes a measure of the strength of assortative mating, for it tells us exactly how much more like its mate one of the pair tends to be than a random sample from the population. It has proved useful in medicine, also, *e. g.*, in investigating the relation between vaccination and immunity or recovery. There are many other ways in which it has, in the short past, been a valuable aid to research, and we confidently believe that the end is not yet.

Unfortunately, most of us, like the immortal Darwin, are 'muzzy . . . on proportion and chance.' It is extremely difficult to apply the formulæ which the mathematicians have worked out, and still harder to understand them. But, undoubtedly, the reward is rich. It is not only that we thereby secure accurate data upon important questions. It is, furthermore, not a 'mere formal clothing of biological conceptions with mathematical symbols.' Such analysis tends to crystallize our notions and makes our conceptions more definite; it will open up new lines of thought, and carry us toward the ultimate goal of a complete understanding of the mechanics of evolution. As in all valuable lines of thought the pitfalls are many, but progress is certain. "Ignoramus; in hoc signo laboremus!"

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[4] Index of abmodality $= \frac{x}{\sigma}$

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DISCUSSION

THE APPLICATION OF CALCULUS TO MENTAL PHENOMENA

IN one of Dr. Montague's recent publications on time perception it was pointed out that we can get a very simple expression for the 'specious present,' which was found to be $\frac{do}{ds}$, if we denote by o the objective and by s the subjective elements of a psychosis. The second derivative would determine the time flow. Without considering the important philosophical results of the theory we shall make the following observations about the method.

The author considers the ratio of the increments Δo and Δs, which occur in the time Δt, and the fraction $\frac{\Delta o}{\Delta s}$ is supposed to approach or attain the limit $\frac{do}{ds}$. It will be of some interest to see what suppositions this statement involves. First of all it is clear that we have to consider the limit of $\frac{\frac{\Delta o}{\Delta t}}{\frac{\Delta s}{\Delta t}}$, because o is not an explicit function of s. Though we know little or nothing about the sufficient conditions of differentiability, we can in this case readily indicate the following necessary conditions: (1) o and s must be continuous; (2) both must have a differential quotient with regard to t; (3) both differential quotients must be continuous; (4) $\frac{ds}{dt}$ must not be zero in the whole time interval under consid-